Quintom evolution in power-law potentials

Emmanuel N. Saridakis*
Department of Physics, University of Athens, GR-15771 Athens, Greece

We investigate quintom evolution in power-law potentials. We extract the early-time, tracker solutions, under the assumption of matter domination. Additionally, we derive analytical solutions at intermediate times, that is at low redshifts, which is the period during the transition from matter to dark-energy domination. A comparison with exact evolution reveals that the tracker solutions are valid within 98% for $z \gtrsim 1.5$, while the intermediate ones are accurate within 98% up to $z \approx 0.5$. Using these expressions we extract two new w-parametrizations, one in terms of the redshift and one in terms of the dark-energy density-parameter, and we present various quintom evolution sub-classes, including quintessence-like or phantom-like cases, realization of the -1-crossing, and non-monotonic w-evolution.

PACS numbers: 95.36.+x, 98.80.-k

I. INTRODUCTION

According to cosmological observations the universe is experiencing an accelerated expansion, and the transition to the accelerated phase has been realized in the recent cosmological past [1]. In order to explain this remarkable behavior, one can modify the theory of gravity [2], or construct "field" models of dark energy. The field models that have been discussed widely in the literature consider a canonical scalar field (quintessence) [3, 4, 5], a phantom field, that is a scalar field with a negative sign of the kinetic term [6], or the combination of quintessence and phantom in a unified model named quintom [7]. Such a combined consideration intends to describe the crossing of the dark-energy equation-of-state parameter w through the phantom divide -1 [8], which is not possible in simple quintessence or simple phantom scenarios where w lies always on the same side of that bound ($w > -1$ for quintessence and $w < -1$ for phantom).

Although one can find potential-independent solutions in field dark energy models [9], in general the cosmological evolution depends significantly on the potential choice. The power-law potential is a widely studied case since, amongst others, it allows for a theoretical justification through supersymmetric considerations [10]. For quintessence [3, 11, 12] or phantom case [13] it has been shown that it exhibits "tracker" behavior, that is for a large class of initial conditions the cosmological evolution converges to a common solution at late times [5], which can be extracted analytically. At intermediate times the dark-energy scalar-field is still small, but non-negligible, and one has to rely on perturbative analytical expressions [12, 13]. However, such an extension to low redshift is very useful, since observations like supernovae Ia, WMAP and SDSS ones, are related to this period [1]. Finally, an additional advantage of these models is that the field energy density remains small at early and intermediate times and thus the known cosmological epochs are not disturbed.

In this work we desire to study the quintom scenario in power-law potentials, both at high and low redshifts, and provide the tracker solutions and the perturbative analytical expressions respectively. Due to the complex nature of the model, we expect that the results will be qualitatively different from simple quintessence and simple phantom models, even at the tracker level. The plan of the work is as follows: In section II we construct the quintom paradigm in power-law potentials and we extract the matter-dominated solutions. In section III we derive the cosmological solutions at intermediate times, that is when dark energy is non-negligible, but still sub-dominant comparing to the matter content of the universe. In section IV we compare our analytical expressions with the exact numerical evolution of the quintom scenario, and in section V we provide some applications of the model, extracting two new w-parametrizations and discussing the cosmological implications. Finally, section VI is devoted to the summary of our results.

*Electronic address: msaridak@phys.uoa.gr

II. EARLY TIME COSMOLOGICAL EVOLUTION: TRACKER SOLUTIONS

We start by constructing the simple quintom cosmological scenario in a flat spacetime. The action of a universe constituted of a canonical ϕ and a phantom σ field, is [7]:

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+V_\phi(\phi)+\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma+V_\sigma(\sigma)+\mathcal{L}_{\rm m}\right], \tag{1}$$

where we have set $8\pi G=1$. $V_\phi(\phi)$ and $V_\sigma(\sigma)$ are respectively the quintessence and phantom field potentials, while the term $\mathcal{L}_{\rm m}$ accounts for the (dark) matter content of the universe, considered as dust. Finally, although we could straightforwardly include baryonic matter and radiation in the model, for simplicity reasons we neglect them since we are interested in $z<20$ era. The Friedmann equations read [7]:

$$H^2=\frac{1}{3}\Big(\rho_m+\rho_\phi+\rho_\sigma\Big), \tag{2}$$

$$\dot{H}=-\frac{1}{2}\Big(\rho_m+\rho_\phi+p_\phi+\rho_\sigma+p_\sigma\Big), \tag{3}$$

and the evolution equations for the canonical and the phantom fields are:

$$\dot{\rho}_\phi+3H(\rho_\phi+p_\phi)=0, \tag{4}$$
$$\dot{\rho}_\sigma+3H(\rho_\sigma+p_\sigma)=0, \tag{5}$$

where $H=\dot{a}/a$ is the Hubble parameter and $a=a(t)$ the scale factor. In these expressions, ρ_m is the dark matter density, and p_ϕ and ρ_ϕ are respectively the pressure and density of the canonical field, while p_σ and ρ_σ are the corresponding quantities for the phantom field. They are given by:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2+V_\phi(\phi) \tag{6}$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2-V_\phi(\phi), \tag{7}$$

$$\rho_\sigma = -\frac{1}{2}\dot{\sigma}^2+V_\sigma(\sigma) \tag{8}$$
$$p_\sigma = -\frac{1}{2}\dot{\sigma}^2-V_\sigma(\sigma). \tag{9}$$

Using these expressions, we can equivalently write the evolution equation in field terms as:

$$\ddot{\phi}+3H\dot{\phi}+\frac{\partial V_\phi(\phi)}{\partial\phi}=0 \tag{10}$$
$$\ddot{\sigma}+3H\dot{\sigma}-\frac{\partial V_\sigma(\sigma)}{\partial\sigma}=0. \tag{11}$$

The equations close by considering the evolution of the matter density, which in the case of dust reads simply $\dot{\rho}_m+3H\rho_m=0$, leading to $\rho_m=\rho_{m0}/a^3$, with ρ_{m0} its value at present.

In quintom scenario, dark energy is attributed to the combination of ϕ and σ. Thus, we can define the dark energy density and pressure as $\rho_{DE}\equiv\rho_\phi+\rho_\sigma$ and $p_{DE}\equiv p_\phi+p_\sigma$. Therefore, the dark energy equation-of-state parameter reads

$$w\equiv\frac{p_{DE}}{\rho_{DE}}=\frac{p_\phi+p_\sigma}{\rho_\phi+\rho_\sigma}. \tag{12}$$

In the present work we study quintom evolution in power-law potentials. That is we consider:

$$V_\phi(\phi)=\kappa_\phi\phi^{-\alpha_\phi} \tag{13}$$
$$V_\sigma(\sigma)=\kappa_\sigma\sigma^{-\alpha_\sigma}, \tag{14}$$

where κ_ϕ and κ_σ are constants with units $m^{4+\alpha_\phi}$ and $m^{4+\alpha_\sigma}$ respectively. Note that we do not restrict the values of α_ϕ and α_σ a priori. However, as we are going to see, energy positivity will force α_ϕ to be positive and α_σ to be negative and bounded, that is potential (13) will be an inverse power-law one while (14) will be of a normal power-law form.

We can express the aforementioned cosmological system using the scale factor a as the independent variable, since it is straightforwardly related to the redshift z which is used in observations. Generalizing [12, 13] we define

$$x_\phi = \frac{\rho_\phi + p_\phi}{2(\rho_m + \rho_\phi)} = \frac{\frac{1}{2}\dot{\phi}^2}{3H^2} = \frac{1}{6}\Big(a\frac{d\phi}{da}\Big)^2, \tag{15}$$

$$x_\sigma = \frac{\rho_\sigma + p_\sigma}{2(\rho_m + \rho_\sigma)} = \frac{-\frac{1}{2}\dot{\sigma}^2}{3H^2} = -\frac{1}{6}\Big(a\frac{d\sigma}{da}\Big)^2, \tag{16}$$

and thus (6),(8) can be written as:

$$\frac{1}{2}\dot{\phi}^2 = \frac{x_\phi}{1-x_\phi}[\rho_m + V_\phi(\phi)], \tag{17}$$

$$-\frac{1}{2}\dot{\sigma}^2 = \frac{x_\sigma}{1-x_\sigma}[\rho_m + V_\sigma(\sigma)]. \tag{18}$$

Therefore, we can simply write:

$$\rho_\phi = \frac{x_\phi\rho_m + V_\phi(\phi)}{1-x_\phi} \tag{19}$$

$$\rho_\sigma = \frac{x_\sigma\rho_m + V_\sigma(\sigma)}{1-x_\sigma} \tag{20}$$

$$p_\phi = \frac{x_\phi\rho_m - V_\phi(\phi)(1-2x_\phi)}{1-x_\phi} \tag{21}$$

$$p_\sigma = \frac{x_\sigma\rho_m - V_\sigma(\sigma)(1-2x_\sigma)}{1-x_\sigma}, \tag{22}$$

and thus for the dark energy equation-of-state parameter we obtain:

$$w = \frac{\frac{x_\phi\rho_m - V_\phi(\phi)(1-2x_\phi)}{1-x_\phi} + \frac{x_\sigma\rho_m - V_\sigma(\sigma)(1-2x_\sigma)}{1-x_\sigma}}{\frac{x_\phi\rho_m + V_\phi(\phi)}{1-x_\phi} + \frac{x_\sigma\rho_m + V_\sigma(\sigma)}{1-x_\sigma}}. \tag{23}$$

Similarly, the first Friedman equation can be written as

$$3H^2 = \frac{\rho_m + V_\phi(\phi) + V_\sigma(\sigma)}{1 - x_\phi - x_\sigma}. \tag{24}$$

Finally, using expressions (19)-(22) and (24) the field evolution equations (10) and (11) become

$$a^2\phi'' + \frac{a\phi'}{2}(5 - 3x_\phi - 3x_\sigma) + \frac{3(1-x_\phi-x_\sigma)}{\rho_m + V_\phi(\phi) + V_\sigma(\sigma)}\Big\{\frac{a\phi'[V_\phi(\phi)+V_\sigma(\sigma)]}{2} + \frac{dV_\phi(\phi)}{d\phi}\Big\} = 0 \tag{25}$$

$$a^2\sigma'' + \frac{a\sigma'}{2}(5 - 3x_\phi - 3x_\sigma) + \frac{3(1-x_\phi-x_\sigma)}{\rho_m + V_\phi(\phi) + V_\sigma(\sigma)}\Big\{\frac{a\sigma'[V_\phi(\phi)+V_\sigma(\sigma)]}{2} - \frac{dV_\sigma(\sigma)}{d\sigma}\Big\} = 0, \tag{26}$$

with prime denoting the derivative with respect to a. These equations are exact and account for the complete dynamics of the quintom scenario.

In this section we desire to present the tracker solutions, that is the initial-condition-independent solutions during the matter-dominated era, i.e at early times. Therefore, we consider $\rho_\phi, |p_\phi| \ll \rho_m$ and $\rho_\sigma, |p_\sigma| \ll \rho_m$, or equivalently $x_\phi \ll 1$ and $|x_\sigma| \ll 1$. Under these approximations, equations (25),(26) are simplified as:

$$a^2\phi''_{(0)} + \frac{5a\phi'_{(0)}}{2} + \frac{3}{\rho_m}\frac{dV_\phi(\phi)}{d\phi}\Big|_{\phi=\phi_{(0)}} = 0 \tag{27}$$

$$a^2\sigma''_{(0)} + \frac{5a\sigma'_{(0)}}{2} - \frac{3}{\rho_m}\frac{dV_\sigma(\sigma)}{d\sigma}\Big|_{\sigma=\sigma_{(0)}} = 0. \tag{28}$$

The zero subscript in parentheses "(0)" denotes just these zeroth-order solutions in terms of x_ϕ,x_σ, or equivalently in terms of the quintessence and phantom energy density, and must not be confused later on with the subscript 0 without parentheses which stands for the present value of a quantity.

Equation (27) can be easily solved analytically in the case of power-law potential (13). The general solution for $\alpha_\phi \neq 0$ (which is the trivial case of a constant potential) is

$$\phi_{(0)} = C_\phi(\alpha_\phi) a^{\frac{3}{2+\alpha_\phi}}\,, \tag{29}$$

where the function $C_\phi(\alpha_\phi)$ is related to the potential parameters through

$$C_\phi(\alpha_\phi) = \Big[\frac{2\alpha_\phi(2+\alpha_\phi)^2\kappa_\phi}{3\rho_{m0}(4+\alpha_\phi)}\Big]^{\frac{1}{2+\alpha_\phi}}. \tag{30}$$

In (29) we have kept only the solution part that remains small (together with its derivative) for small a's, in order to be consistent with the matter-dominated approximation ($\rho_\phi, |p_\phi| \ll \rho_m$). This is the reason for the absence of initial-condition dependent constants, which is just the central idea of tracker solutions. In addition, it is easy to see that this solution remains regular for $\alpha_\phi \to -2$. The solution for $\alpha_\phi = -2$ can be shown to diverge at small a and thus we do not write it explicitly.

Similarly, the general solution of equation (28) under potential (14)) is

$$\sigma_{(0)} = C_\sigma(\alpha_\sigma) a^{\frac{3}{2+\alpha_\sigma}}\,, \tag{31}$$

where

$$C_\sigma(\alpha_\sigma) = \Big[-\frac{2\alpha_\sigma(2+\alpha_\sigma)^2\kappa_\sigma}{3\rho_{m0}(4+\alpha_\sigma)}\Big]^{\frac{1}{2+\alpha_\sigma}}. \tag{32}$$

The zeroth-order solution for ρ_ϕ can be calculated from (19) under $x_\phi \ll 1$, that is $\rho_{\phi(0)} = x_{\phi(0)}\rho_m + V_\phi(\phi_{(0)})$, where $x_{\phi(0)} = (a\phi'_{(0)})^2/6$. The result is:

$$\rho_{\phi(0)} = 3\rho_{m0}\frac{[C_\phi(\alpha_\phi)]^2}{\alpha_\phi(2+\alpha_\phi)}\,a^{-\frac{3\alpha_\phi}{2+\alpha_\phi}}. \tag{33}$$

Furthermore, using the zeroth-order approximation of (20), i.e $\rho_{\sigma(0)} = x_{\sigma(0)}\rho_m + V_\sigma(\sigma_{(0)})$ with $x_{\sigma(0)} = -(a\sigma'_{(0)})^2/6$, we obtain

$$\rho_{\sigma(0)} = -3\rho_{m0}\frac{[C_\sigma(\alpha_\sigma)]^2}{\alpha_\sigma(2+\alpha_\sigma)}\,a^{-\frac{3\alpha_\sigma}{2+\alpha_\sigma}}. \tag{34}$$

Similarly to the simple phantom case [13], expression (34) and the requirement of phantom-energy positivity leads to the constraint $-2 < \alpha_\sigma < 0$, that is the phantom potential is a normal power law. Additionally, (33) leads to $\alpha_\phi < -2$ or $\alpha_\phi > 0$, with the second case being physically more interesting since it corresponds to the well-studied inverse power-law potential of the literature [3, 11, 12].

Finally, using the zeroth-order approximation of (21) and (22), namely $p_{\phi(0)} = x_{\phi(0)}\rho_m - V_\phi(\phi_{(0)})$ and $p_{\sigma(0)} = x_{\sigma(0)}\rho_m - V_\sigma(\sigma_{(0)})$ respectively, we obtain

$$p_{\phi(0)} = -6\rho_{m0}\frac{[C_\phi(\alpha_\phi)]^2}{\alpha_\phi(2+\alpha_\phi)^2}\,a^{-\frac{3\alpha_\phi}{2+\alpha_\phi}} \tag{35}$$

$$p_{\sigma(0)} = 6\rho_{m0}\frac{[C_\sigma(\alpha_\sigma)]^2}{\alpha_\sigma(2+\alpha_\sigma)^2}\,a^{-\frac{3\alpha_\sigma}{2+\alpha_\sigma}}. \tag{36}$$

At this stage, the zeroth-order, that is early-time, results coincide with those of simple quintessence [12] or simple phantom models [13]. This was expected since the exact equations of motion (25),(26), which correspond to a coupled system for the two fields, under the zeroth-order approximation lead to the un-coupled equation system (27),(28). Thus, at early times the two fields evolve independently. However, even at this approximation level, the complexity of the quintom model leads to qualitatively different results. Indeed, dark energy is attributed to both fields under definition (12). Therefore, we acquire:

$$w_{(0)} = \frac{p_{\phi(0)} + p_{\sigma(0)}}{\rho_{\phi(0)} + \rho_{\sigma(0)}} = \frac{-2\frac{[C_\phi(\alpha_\phi)]^2}{\alpha_\phi(2+\alpha_\phi)^2}\,a^{-\frac{3\alpha_\phi}{2+\alpha_\phi}} + 2\frac{[C_\sigma(\alpha_\sigma)]^2}{\alpha_\sigma(2+\alpha_\sigma)^2}\,a^{-\frac{3\alpha_\sigma}{2+\alpha_\sigma}}}{\frac{[C_\phi(\alpha_\phi)]^2}{\alpha_\phi(2+\alpha_\phi)}\,a^{-\frac{3\alpha_\phi}{2+\alpha_\phi}} - \frac{[C_\sigma(\alpha_\sigma)]^2}{\alpha_\sigma(2+\alpha_\sigma)}\,a^{-\frac{3\alpha_\sigma}{2+\alpha_\sigma}}}. \tag{37}$$

As we observe, although in the total absence of one of the fields this expression gives the corresponding results of simple quintessence ($w_{(0)} = -\frac{2}{2+\alpha_\phi}$ [12]) and simple phantom ($w_{(0)} = -\frac{2}{2+\alpha_\sigma}$ [13]), when both fields are present and despite the fact that they are not coupled at this zeroth-level, the behavior of $w_{(0)}$ is qualitative different. In particular, as can be clearly seen, $w_{(0)}$ is not a constant but it is a-dependent although it is the tracker solution. On the contrary, in simple quintessence and simple phantom scenarios one should go beyond tracker solutions, that is to intermediate times, in order to reveal a varying equation-of-state parameter. This feature makes the model at hand cosmologically interesting. We mention that according to the specific model and choice of parameters, $w_{(0)}$ can evolve below or above -1, or even experience the phantom-divide crossing.

Finally, we can calculate the zeroth-order behavior of the dark-energy density parameter as:

$$\Omega_{DE(0)} = \frac{\rho_{\phi(0)} + \rho_{\sigma(0)}}{\rho_m} = 3\frac{[C_\phi(\alpha_\phi)]^2}{\alpha_\phi(2+\alpha_\phi)}\, a^{\frac{6}{2+\alpha_\phi}} - 3\frac{[C_\sigma(\alpha_\sigma)]^2}{\alpha_\sigma(2+\alpha_\sigma)}\, a^{\frac{6}{2+\alpha_\sigma}}, \tag{38}$$

where we have used that $\rho_m = \rho_{m0}/a^3$.

Expressions (29)-(38) are the tracker solutions for quintom cosmology in power-law potentials. Equivalently they can be expressed as a function of time, considering $a \propto t^{2/3}$ since we are in the matter-dominated era, or as a function of the redshift z through $\frac{a_0}{a} = 1+z$, with $a_0 = 1$ the present value. They provide an excellent approximation to the behavior of the quintom scenario as long as $\rho_\phi, |p_\phi| \ll \rho_m$ and $\rho_\sigma, |p_\sigma| \ll \rho_m$, that is at early times.

III. COSMOLOGICAL EVOLUTION AT INTERMEDIATE TIMES

As time passes and cosmological evolution continues, the scalar fields increase and dark-energy density becomes non-negligible, although still dominated by the dark-matter one. Therefore, we expect that progressively the various quantities will start diverging from the expressions obtained above. We are interested in the analytical investigation of this intermediate evolution stage, and thus we perform a first-order perturbation to the zeroth-order solutions of section II. The obtained solutions will be significant for the period during the transition from matter to dark-energy domination. The corresponding quantities are denoting by the subscript "(1)", and the total ones by tilde, i.e:

$$\widetilde{\phi} = \phi_{(0)} + \phi_{(1)},\ \widetilde{\sigma} = \sigma_{(0)} + \sigma_{(1)},\ \widetilde{\rho}_\phi = \rho_{\phi(0)} + \rho_{\phi(1)},\ \widetilde{\rho}_\sigma = \rho_{\sigma(0)} + \rho_{\sigma(1)},\ \widetilde{w} = w_{(0)} + w_{(1)}. \tag{39}$$

The aforementioned perturbations can be equivalently considered as keeping terms up to first order in x_ϕ and x_σ (i.e keeping only $x_{\phi(0)}$ and $x_{\sigma(0)}$) and in $\phi_{(1)}/\phi_{(0)}$ and $\sigma_{(1)}/\sigma_{(0)}$, in the exact evolution equations (25),(26). An additional convenient formula can arise form the expansions of the potentials as $V_\phi(\widetilde{\phi}) = V_\phi(\phi_{(0)}) + \frac{dV_\phi(\phi_{(0)})}{d\phi_{(0)}}\phi_{(1)} + \mathcal{O}(\phi_{(1)}^2)$, and similarly for $V_\sigma(\widetilde{\sigma})$. Substituting the expansions (39) into (25),(26), we acquire the approximate evolution equations at this intermediate-time region:

$$a^2\phi''_{(1)} + \frac{a}{2}\left\{5\phi'_{(1)} - 3[x_{\phi(0)} + x_{\sigma(0)}]\phi'_{(0)}\right\} + \frac{3a\phi'_{(0)}[V_\phi(\phi) + V_\sigma(\sigma)]}{2\rho_m} - \frac{3[\rho_{\phi(0)} + \rho_{\sigma(0)}]}{{\rho_m}^2}\frac{dV_\phi(\phi_{(0)})}{d\phi_{(0)}} + \frac{3}{\rho_m}\frac{d^2V_\phi(\phi_{(0)})}{d\phi_{(0)}^2}\phi_{(1)} = 0, \tag{40}$$

$$a^2\sigma''_{(1)} + \frac{a}{2}\left\{5\sigma'_{(1)} - 3[x_{\phi(0)} + x_{\sigma(0)}]\sigma'_{(0)}\right\} + \frac{3a\sigma'_{(0)}[V_\phi(\phi) + V_\sigma(\sigma)]}{2\rho_m} + \frac{3[\rho_{\phi(0)} + \rho_{\sigma(0)}]}{{\rho_m}^2}\frac{dV_\sigma(\sigma_{(0)})}{d\sigma_{(0)}} - \frac{3}{\rho_m}\frac{d^2V_\sigma(\sigma_{(0)})}{d\sigma_{(0)}^2}\sigma_{(1)} = 0, \tag{41}$$

where $x_{\phi(0)} = (a\phi'_{(0)})^2/6$ and $x_{\sigma(0)} = -(a\sigma'_{(0)})^2/6$. Inserting the explicit power-law potential forms (13) and (14) we finally obtain

$$a^2\phi''_{(1)} + \frac{a}{2}\left[5\phi'_{(1)} - \frac{a^2}{2}\,\phi'^3_{(0)}\right] + \frac{a^3}{4}\,\sigma'^2_{(0)}\phi'_{(0)} + \frac{27\kappa_\phi a^3\phi_{(0)}^{1-\alpha_\phi}}{2(2+\alpha_\phi)\rho_{m0}} + \frac{3\left[\kappa_\sigma\sigma_{(0)}^{-\alpha_\sigma} - \frac{\rho_{m0}}{6a}\sigma'^2_{(0)}\right]a^6}{\rho_{m0}^2}\kappa_\phi\alpha_\phi\phi_{(0)}^{-1-\alpha_\phi} + $$
$$+\frac{3a^4}{2\rho_{m0}}\kappa_\sigma\sigma_{(0)}^{-\alpha_\sigma}\phi'_{(0)} + \frac{3\alpha_\phi(1+\alpha_\phi)a^3\kappa_\phi\phi_{(0)}^{-2-\alpha_\phi}\phi_{(1)}}{\rho_{m0}} = 0 \tag{42}$$

and

$$a^2\sigma''_{(1)} + \frac{a}{2}\left[5\sigma'_{(1)} + \frac{a^2}{2}\,\sigma'^3_{(0)}\right] - \frac{a^3}{4}\,\phi'^2_{(0)}\sigma'_{(0)} + \frac{27\kappa_\sigma a^3\sigma_{(0)}^{1-\alpha_\sigma}}{2(2+\alpha_\sigma)\rho_{m0}} - \frac{3\left[\kappa_\phi\phi_{(0)}^{-\alpha_\phi} + \frac{\rho_{m0}}{6a}\phi'^2_{(0)}\right]a^6}{\rho_{m0}^2}\kappa_\sigma\alpha_\sigma\sigma_{(0)}^{-1-\alpha_\sigma} + $$
$$+\frac{3a^4}{2\rho_{m0}}\kappa_\phi\phi_{(0)}^{-\alpha_\phi}\sigma'_{(0)} - \frac{3\alpha_\sigma(1+\alpha_\sigma)a^3\kappa_\sigma\sigma_{(0)}^{-2-\alpha_\sigma}\sigma_{(1)}}{\rho_{m0}} = 0. \tag{43}$$

Despite their complicated form, these equation allow for a relatively simple general solution, if we keep only the part that remains small (together with its derivative) for small a's, in order to be consistent with the matter-dominated approximation. For $\alpha_\phi > 0$ and $-2 < \alpha_\sigma < 0$ it reads:

$$\phi_{(1)} = B_\phi(\alpha_\phi, \alpha_\sigma)\, a^{\frac{9}{2+\alpha_\phi}}, \tag{44}$$

$$\sigma_{(1)} = B_\sigma(\alpha_\phi, \alpha_\sigma)\, a^{\frac{9}{2+\alpha_\sigma}}, \tag{45}$$

where the functions $B_\phi(\alpha_\phi, \alpha_\sigma)$ and $B_\sigma(\alpha_\phi, \alpha_\sigma)$ are related to the potential parameters through

$$B_\phi(\alpha_\phi, \alpha_\sigma) = 3C_\phi(\alpha_\phi)\frac{\{-[C_\phi(\alpha_\phi)]^2\alpha_\sigma(2+\alpha_\sigma)^2(6+\alpha_\phi) + [C_\sigma(\alpha_\sigma)]^2\alpha_\phi(2+\alpha_\phi)[12+4(\alpha_\phi+\alpha_\sigma)+\alpha_\phi\alpha_\sigma]\}}{\alpha_\phi(2+\alpha_\phi)\alpha_\sigma(2+\alpha_\sigma)^2(28+8\alpha_\phi+\alpha_\phi^2)} \tag{46}$$

$$B_\sigma(\alpha_\phi, \alpha_\sigma) = -3C_\sigma(\alpha_\sigma)\frac{\{-[C_\sigma(\alpha_\sigma)]^2\alpha_\phi(2+\alpha_\phi)^2(6+\alpha_\sigma) + [C_\phi(\alpha_\phi)]^2\alpha_\sigma(2+\alpha_\sigma)[12+4(\alpha_\phi+\alpha_\sigma)+\alpha_\phi\alpha_\sigma]\}}{\alpha_\sigma(2+\alpha_\sigma)\alpha_\phi(2+\alpha_\phi)^2(28+8\alpha_\sigma+\alpha_\sigma^2)}, \tag{47}$$

(that is $B_\sigma(\alpha_\phi, \alpha_\sigma)$ arises form $B_\phi(\alpha_\phi, \alpha_\sigma)$ changing the index ϕ to σ and vice versa, and adding an overall minus sign). In these expressions the complexity of the quintom model becomes clear, since they are qualitatively different from those of simple quintessence [12] or simple phantom [13] models. Indeed, since the first-order approximated evolution equations (42),(43) are coupled, their general solution reveals a strong interdependence of the two fields, too. This feature will become even stronger in the $w_{(1)}$-solution below, which by definition it already depends on both fields. Finally, in the total absence of one field, expressions (46), (47) give exactly those of the corresponding simple models [12, 13].

The perturbation of ρ_ϕ can be calculated from (19) keeping the corresponding terms, thus:

$$\rho_{\phi(1)} = x_{\phi(0)}\rho_{\phi(0)} + x_{\phi(1)}\rho_m - \alpha_\phi\frac{\phi_{(1)}}{\phi_{(0)}}V_\phi(\phi), \tag{48}$$

where $x_{\phi(1)} = \frac{1}{3}a^2\phi'_{(0)}\phi'_{(1)}$ as it arises from $\widetilde{x}_\phi = x_{\phi(0)} + x_{\phi(1)} = a^2(\phi'^2_{(0)} + 2\phi'_{(0)}\phi'_{(1)})/6$. Therefore, using also (29), (33) and (44) in order to express the result in terms of $\rho_{\phi(0)}$, we obtain

$$\rho_{\phi(1)} = \Big\{\frac{3[C_\phi(\alpha_\phi)]^3 - B_\phi(\alpha_\phi,\alpha_\sigma)\alpha_\phi(\alpha_\phi^2-4)}{2C_\phi(\alpha_\phi)(2+\alpha_\phi)^2}\Big\}\rho_{\phi(0)}\, a^{\frac{6}{2+\alpha_\phi}}. \tag{49}$$

Repeating the same procedure for ρ_σ we acquire:

$$\rho_{\sigma(1)} = -\Big\{\frac{3[C_\sigma(\alpha_\sigma)]^3 + B_\sigma(\alpha_\phi,\alpha_\sigma)\alpha_\sigma(\alpha_\sigma^2-4)}{2C_\sigma(\alpha_\sigma)(2+\alpha_\sigma)^2}\Big\}\rho_{\sigma(0)}\, a^{\frac{6}{2+\alpha_\sigma}}. \tag{50}$$

The perturbation of w can arise form (12) as

$$w_{(1)} = \frac{p_{\phi(1)} + p_{\sigma(1)}}{\rho_{\phi(0)} + \rho_{\sigma(0)}} - \frac{p_{\phi(0)} + p_{\sigma(0)}}{[\rho_{\phi(0)} + \rho_{\sigma(0)}]^2}\,[\rho_{\phi(1)} + \rho_{\sigma(1)}], \tag{51}$$

with $p_{\phi(1)} = x_{\phi(0)}\rho_{\phi(0)} + x_{\phi(1)}\rho_m + \alpha_\phi\frac{\phi_{(1)}}{\phi_{(0)}}V_\phi(\phi)$ (and similarly for $p_{\sigma(1)}$). The final result reads

$$w_{(1)} = \frac{\alpha_\phi^2(2+\alpha_\phi^2)\alpha_\sigma^2(2+\alpha_\sigma^2)a^{\frac{12(\alpha_\phi+\alpha_\sigma+\alpha_\phi\alpha_\sigma)}{(2+\alpha_\phi)(2+\alpha_\sigma)}}}{2\Big\{[C_\sigma(\alpha_\sigma)]^2\alpha_\phi(2+\alpha_\phi)a^{\frac{3\alpha_\phi}{2+\alpha_\phi}} - [C_\phi(\alpha_\phi)]^2\alpha_\sigma(2+\alpha_\sigma)a^{\frac{3\alpha_\sigma}{2+\alpha_\sigma}}\Big\}^2}\,(d_1+d_2+d_3+d_4), \tag{52}$$

where

$$d_1 = \frac{[C_\phi(\alpha_\phi)]^3(4+\alpha_\phi)\{3[C_\phi(\alpha_\phi)]^3 + B_\phi(\alpha_\phi,\alpha_\sigma)\alpha_\phi(2+\alpha_\phi)(6+\alpha_\phi)\}}{\alpha_\phi^2(2+\alpha_\phi)^5}a^{\frac{6(1-\alpha_\phi)}{2+\alpha_\phi}}$$

$$d_2 = \frac{[C_\sigma(\alpha_\sigma)]^3(4+\alpha_\sigma)\{-3[C_\sigma(\alpha_\sigma)]^3 + B_\sigma(\alpha_\phi,\alpha_\sigma)\alpha_\sigma(2+\alpha_\sigma)(6+\alpha_\sigma)\}}{\alpha_\sigma^2(2+\alpha_\sigma)^5}a^{\frac{6(1-\alpha_\sigma)}{2+\alpha_\sigma}}$$

$$d_3 = -\frac{C_\phi(\alpha_\phi)[C_\sigma(\alpha_\sigma)]^2\{3(4+\alpha_\sigma)[C_\phi(\alpha_\phi)]^3 + B_\phi(\alpha_\phi,\alpha_\sigma)\alpha_\phi(2+\alpha_\phi)[24+(10+\alpha_\phi)\alpha_\sigma]\}}{\alpha_\phi(2+\alpha_\phi)^3\alpha_\sigma(2+\alpha_\sigma)^2}\,a^{\frac{6}{2+\alpha_\sigma}-\frac{6\alpha_\phi}{2+\alpha_\phi}}$$

$$d_4 = -\frac{C_\sigma(\alpha_\sigma)[C_\phi(\alpha_\phi)]^2\{-3(4+\alpha_\phi)[C_\sigma(\alpha_\sigma)]^3 + B_\sigma(\alpha_\phi,\alpha_\sigma)\alpha_\sigma(2+\alpha_\sigma)[24+(10+\alpha_\sigma)\alpha_\phi]\}}{\alpha_\sigma(2+\alpha_\sigma)^3\alpha_\phi(2+\alpha_\phi)^2}\,a^{\frac{6}{2+\alpha_\phi}-\frac{6\alpha_\sigma}{2+\alpha_\sigma}}.$$

Note that expressions (49), (50) and (52) in the total absence of one field, coincide with those of the corresponding simple quintessence [12] and simple phantom [13] models. The qualitative difference of quintom scenario is also reflected in the un-specified sign of the correction terms, contrary to simple models where these corrections have a constant sign ($\phi_{(1)}$ is always negative in both quintessence and phantom cases, and $\rho_{\phi(1)} > 0$ and $w_{\phi(1)} < 0$ in simple quintessence while $\rho_{\phi(1)} < 0$ and $w_{\phi(1)} > 0$ in simple phantom scenario).

In summary, inserting (44), (45), (49), (50) and (52) in the perturbative expansion (39) we finally obtain the analytical solutions for intermediate times, namely $\widetilde{\phi}, \widetilde{\sigma}, \widetilde{\rho}_\phi, \widetilde{\rho}_\sigma$ and $\widetilde{w}$. Finally, $\widetilde{\Omega}_{DE}$ can be calculated as $\widetilde{\Omega}_{DE} = (\widetilde{\rho}_\phi + \widetilde{\rho}_\sigma)/(\widetilde{\rho}_\phi + \widetilde{\rho}_\sigma + \rho_m)$. We mention that since we have obtained the aforementioned quantities as a function of the scale factor, it is straightforward to express them as a function of the redshift z through $\frac{1}{a} = 1 + z$.

IV. COMPARING ANALYTICAL AND NUMERICAL RESULTS

In the previous two sections we extracted analytical expressions for the quintom scenario in general power-law potentials, when the universe is dominated by the dark matter sector. Concerning their applicability, we argued that these solutions are valid at early and intermediate times, that is at both high and low redshift. In order to examine the precision of our results, we evolve numerically the exact cosmological system calculating the exact quantities $\phi(a)$, $\Omega_{DE}(a)$, $w(a)$, and then we study their divergence from the zeroth and first order formulae derived above. This can be achieved by examining the ratios $\phi_{(0)}/\phi$, $\widetilde{\phi}/\phi$, $\sigma_{(0)}/\sigma$, $\widetilde{\sigma}/\sigma$ and similarly $\Omega_{DE(0)}/\Omega_{DE}$, $\widetilde{\Omega}_{DE}/\Omega_{DE}$ and $w_{(0)}/w$, $\widetilde{w}/w$. The initial conditions are chosen consistently with initial matter domination, since if this requirement is fulfilled then the results do not depend on the specific initial conditions, as expected. Finally, we fix κ_ϕ and κ_σ in order to acquire $\Omega_{m0} \approx 0.28$ and $\Omega_{DE0} \approx 0.72$ at present. Note that contrary to simple quintessence and simple phantom cases, one can satisfy these observational requirements by many different realizations of the quintom scenario (i.e many different potential-parameter choices), which is an additional advantage of the model at hand.

In fig. 1 we present the ratios of the zeroth and first order field solutions to the exact numerical values, as a function of the redshift. The calculations have been performed for two potential combinations, namely $\alpha_\phi = 2, \alpha_\sigma = -1$ and

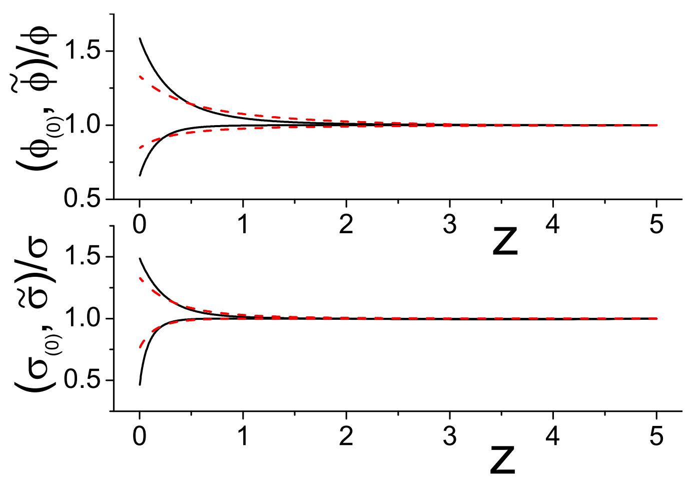

FIG. 1: (Color Online) *Upper graph: The ratios $\phi_{(0)}/\phi$ (upper two curves) and $\widetilde{\phi}/\phi$ (lower two curves) as a function of the redshift, for $\alpha_\phi = 2, \alpha_\sigma = -1$ (black, solid) and $\alpha_\phi = 1, \alpha_\sigma = -0.5$ (red, dashed). Lower graph: The ratios $\sigma_{(0)}/\sigma$ (upper two curves) and $\widetilde{\sigma}/\sigma$ (lower two curves) as a function of the redshift, for $\alpha_\phi = 2, \alpha_\sigma = -1$ (black, solid) and $\alpha_\phi = 1, \alpha_\sigma = -0.5$ (red, dashed).*

$\alpha_\phi = 1, \alpha_\sigma = -0.5$. As we observe, $\phi_{(0)}/\phi$ and $\sigma_{(0)}/\sigma$ are very close to 1 for $z \gtrsim 1.5$, that is the tracker solution is a very good approximation at this early evolution stage, with errors of less than 3%. Furthermore, for $1.5 \gtrsim z \gtrsim 0.5$ the zeroth-order solution is not a good approximation (with error 15%) but the first-order one remains within 95% accuracy. As expected these errors are larger than the corresponding results of simple quintessence [12] and simple phantom [13] models, since the exact system is coupled and thus the evolution depends on both fields. Finally, at late times, that is for $z < 0.5$, the scalar fields are enhanced significantly, dark energy dominates the universe, and our approximation breaks down rapidly.

In fig. 2 we depict $\Omega_{DE(0)}/\Omega_{DE}$ and $\widetilde{\Omega}_{DE}/\Omega_{DE}$ as a function of z. The divergence of the zeroth-order solution

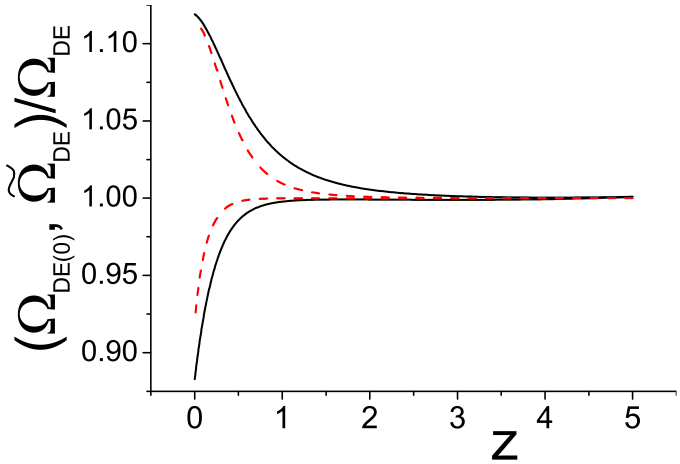

FIG. 2: (Color Online) *The ratios $\Omega_{DE(0)}/\Omega_{DE}$ (upper two curves) and $\widetilde{\Omega}_{DE}/\Omega_{DE}$ (lower two curves) as a function of the redshift, for $\alpha_\phi = 2, \alpha_\sigma = -1$ (black, solid) and $\alpha_\phi = 1, \alpha_\sigma = -0.5$ (red, dashed).*

from the exact one starts at $z \approx 1.5$. However, we can see that the first-order solution is very accurate (with error less than 2%) up to $z \approx 0.5$. After that point, the first-order solution starts diverging rapidly from the exact one and our approximation is not valid.

Finally, in fig. 3 we present $w_{(0)}/w$ and $\widetilde{w}/w$ as a function of the redshift. We observe that the tracker solution is

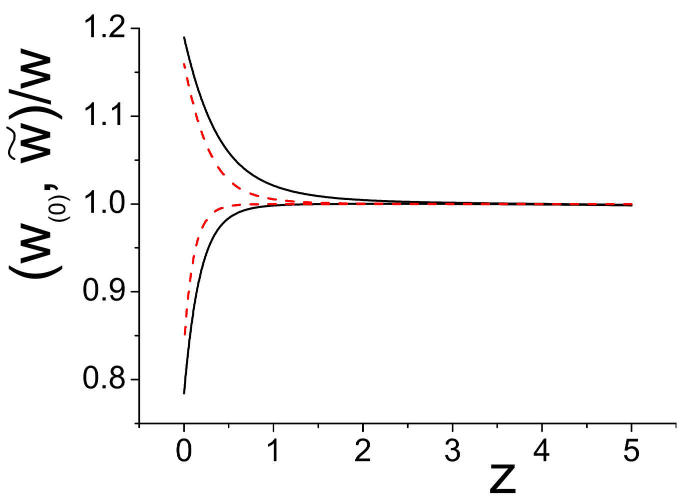

FIG. 3: (Color Online) *The ratios $w_{(0)}/w$ (upper two curves) and $\widetilde{w}/w$ (lower two curves) as a function of the redshift, for $\alpha_\phi = 2, \alpha_\sigma = -1$ (black, solid) and $\alpha_\phi = 1, \alpha_\sigma = -0.5$ (red, dashed).*

very accurate up to $z \approx 1.5$, while the first-order one agrees with the exact evolution within 2% up to $z \approx 0.5$. As expected these errors are larger than the corresponding results of simple models [12, 13], since the error of w depends on the errors of both fields.

Therefore, from these tree figures we conclude that the tracker, that is the zeroth-order, solution is accurate within an error of 2% at early times and up to $z \approx 1.5$. At intermediate times, that is at low redshifts, we have to use the first-order analytical solution, which agrees with the exact cosmological evolution up to $z \approx 0.5$, within an error of 2%. After that point, the dark energy sector enhances, it dominates the dark matter one, and our approximation breaks down rapidly.

V. COSMOLOGICAL IMPLICATIONS AND DISCUSSION

Having tested the accuracy of our analytical expressions and having determined their applicability region, we can use them to describe an arbitrary quintom evolution in power-law potentials up to $z = 0.5$. Clearly, the tracker solutions are much simpler and can be used as a first approximation. Contrary to simple quintessence and simple phantom models, w presents a varying behavior even at this zeroth-order approximation level, and thus it can offer a reliable description of the system (quantitatively up to $z \approx 1.5$ and qualitatively later on). For intermediate times one can use at will the analytical expressions for $\widetilde{w}$ and $\widetilde{\Omega}_{DE}$, which are very accurate up to $z \approx 0.5$. Here we present an additional simplification which allows for an easier application. Observing that their complicated form is due to the a-independent coefficients (which are determined by the potential parameters) rather than the exponents of a, we can make an easy shortening keeping the most significant a-terms and thus resulting to a convenient simulation of w-evolution. Therefore, expressing the formula in terms of the redshift z through $\frac{1}{a} = 1 + z$, we finally acquire:

$$w_{fit\,\mathrm{z}}(z) = w_1(1+z)^{s_1} + (w_0 - w_1)(1+z)^{s_2}. \tag{53}$$

In this w-parametrization w_0 is its present value, which can be taken from observations, while w_1, s_1 and s_2 are determined by the potential exponents in a rather complicated way. Equivalently, we can consider w_1, s_1 and s_2 as additional, free parameters. In this way, the relative large number of parameters in the fit (53) is attributed to the general behavior of quintom scenario, which must be able to present a large class of cosmological evolutions. Furthermore, the use of three or four parameters in w-parametrizations is usual in the literature [14].

Let us examine the accuracy of the fit (53), comparing it with the exact cosmological evolution of the model at hand. In fig. 4 we depict the ratio $w_{fit\,\mathrm{z}}/w$ as a function of z for two potential choices. In this figure the value w_0 has been determined numerically, that is why $w_{fit\,\mathrm{z}}$ and w coincide both initially and finally. As we observe, the proposed

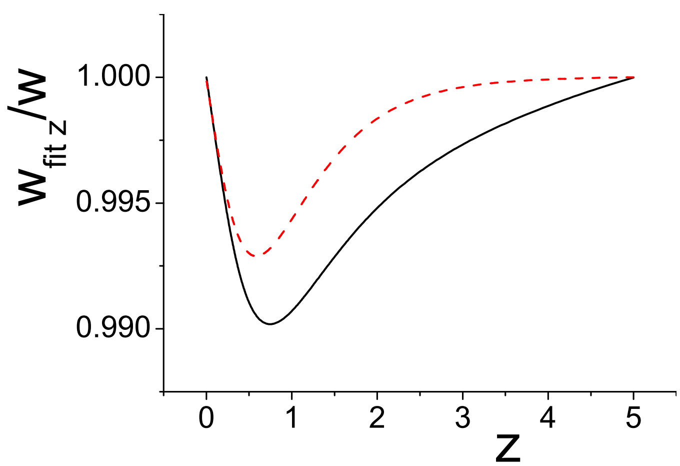

FIG. 4: (Color Online) *The ratio $w_{fit\,z}/w$, where $w_{fit\,z}$ is given by (53), as a function of z, for $\alpha_\phi = 2, \alpha_\sigma = -1$ (black, solid) and $\alpha_\phi = 1, \alpha_\sigma = -0.5$ (red, dashed).*

w-parametrization, given by (53), agrees with the exact evolution within 99%. Similarly to the figures of the previous section, the errors are larger for larger α_ϕ and $|\alpha_\sigma|$, however small α_ϕ, $|\alpha_\sigma|$ are expected to be more realistic. The very small error is due to the general and multi-parametric form of the fit (53), which allows for a very satisfactory fit of a general quintom evolution in power-law potentials.

Another useful approximated relation, that can arise from our analytical results, is the expression of w as a function of Ω_{DE}. Indeed, since we have derived the relations $w(a)$ and $\Omega_{DE}(a)$, we can keep the most significant a-terms and then eliminate a, resulting to $w(\Omega_{DE})$. Doing so we obtain:

$$w_{fit\Omega_{DE}}(\Omega_{DE}) = w_2 + w_3\Omega_{DE}^{s_3} + w_4\Omega_{DE}^{s_4}. \tag{54}$$

The various parameters can be determined by the specific potential choice in a rather complicated way. Equivalently we can consider them as free parameters, suitably fitted in order to acquire the best agreement with the exact cosmological evolution. Using a χ^2 minimization routine to determine the best-fit values of these parameters, in fig. 5 we present the ratio $w_{fit\Omega_{DE}}/w$ as a function of Ω_{DE} for two potential choices, where the present Ω_{DE}-value has

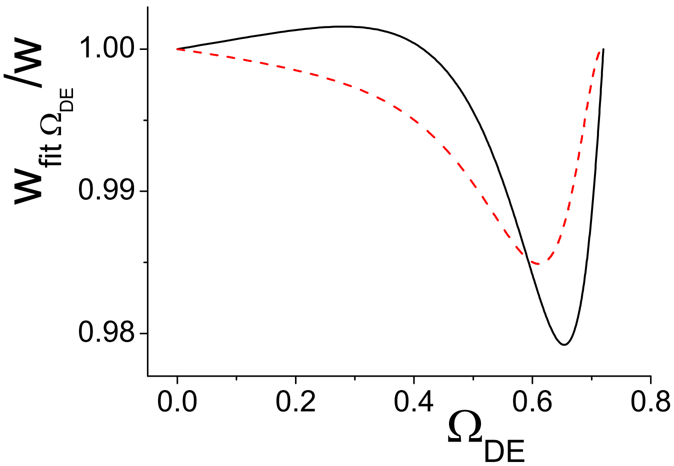

FIG. 5: (Color Online) *The ratio $w_{fit\Omega_{DE}}/w$, where $w_{fit\Omega_{DE}}$ is given by (54), as a function of Ω_{DE}, for $\alpha_\phi = 2, \alpha_\sigma = -1$ (black, solid) and $\alpha_\phi = 1, \alpha_\sigma = -0.5$ (red, dashed).*

been taken to be 0.72, both in $w_{fit\Omega_{DE}}$ and w. The error introduced by the fit (54) is less than 2.5% for the examined potential cases, mainly at late times. The fact that it is larger than the corresponding error of $w_{fit\,z}$, is a result of the simplifications we performed in order to eliminate a and obtain $w_{fit\Omega_{DE}}(\Omega_{DE})$. Finally, we stress that fits (53) and (54) can be considered to be valid up to present epoch ($z = 0$ and $\Omega_{DE} \approx 0.72$) although they have arisen from expressions valid up to $z \approx 0.5$, since the relative large number of free parameters leads to small errors and one can always adjust the fit to coincide with the exact evolution both initially and finally.

Since we have examined the accuracy of our $w(z)$ and $w(\Omega_{DE})$ parametrizations, we can use them at will to describe a large class of cosmological evolutions. In fig. 6 we depict $w_{fit\,z}(z)$ for four different cosmological cases. Case I corresponds to a quintessence-like evolution, where w remains always larger than -1. Case II corresponds to

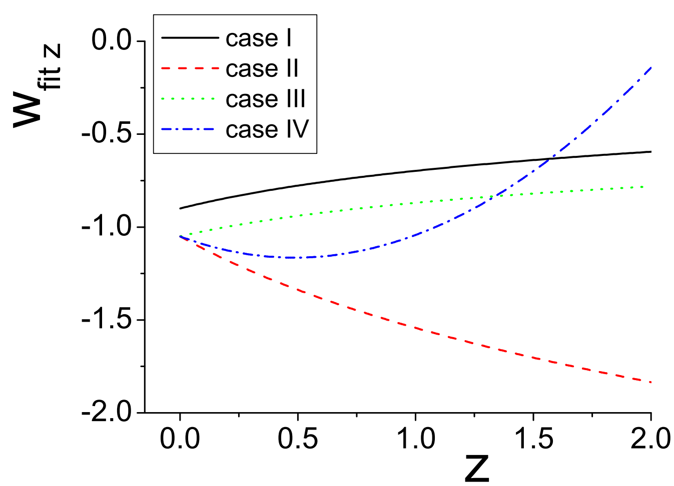

FIG. 6: (Color Online)$w_{fit\,z}(z)$ *for four different cosmological cases, as it arises from parametrization (53). Case I: $w_0 = -0.9$, $w_1 = -1$, $s_1 = -1/3$, $s_2 = 1/5$ (black, solid), case II: $w_0 = -1.05$, $w_1 = 1$, $s_1 = -1/3$, $s_2 = 1/5$ (red, dashed), case III: $w_0 = -1.05$, $w_1 = -1$, $s_1 = -1/3$, $s_2 = 1/5$ (green, dotted), case IV: $w_0 = -1.05$, $w_1 = -1.8$, $s_1 = 1$, $s_2 = 1.8$ (blue, dashed-dotted).*

a phantom-like evolution, with $w < -1$. Case III corresponds to a quintom evolution where w crosses the phantom divide in the recent cosmological past, as might be suggested by observations within 95% confidence level [1]. It is interesting to see that (53) allows for a non-monotonic $w_{fit\,z}(z)$, too. Indeed, case IV corresponds to such a behavior, where w experiences the -1-crossing from above to below and then it starts increasing again.

In summary, as was mentioned above, expression (53) can describe a very large class of cosmological evolutions.

Similar results can be obtained using $w_{fit\Omega_{DE}}(\Omega_{DE})$ from (54), where one can acquire various $w(\Omega_{DE})$ cosmological behaviors. Finally, it is easy to see, that the case of simple quintessence can be described by (53) and (54) using $s_1 = 0$, $s_2 = -\frac{6}{2+\alpha_\phi}$, $s_3 = 1$, $w_1 = w_2 = -\frac{2}{2+\alpha_\phi}$, $w_3 = -\frac{2\alpha_\phi(4+\alpha_\phi)}{(2+\alpha_\phi)(\alpha_\phi^2+8\alpha_\phi+28)}$, $w_4 = 0$, while for the simple phantom model the corresponding parameters are the same with $\alpha_\phi \rightarrow \alpha_\sigma$. However, it is more convenient to consider them as free parameters.

VI. CONCLUSIONS

In this work we studied quintom models with power-law potentials. Firstly, we extracted the tracker solutions under the assumption of matter domination. These solutions correspond to the general and common behavior of all such models at early times, that is at high redshifts. Contrary to simple quintessence and simple phantom cases, even at this zeroth-order approximation level, w is not constant but evolving, which is a result of the complex nature of quintom scenario. The tracker solutions are quantitatively valid up to $z \approx 1.5$ (within 98% accuracy), but since this zeroth-order approximation possesses the qualitative features of the exact solution (varying and not constant w), it could be used up to low redshifts in order to provide a first picture of the corresponding cosmological evolution.

In addition, we extracted the general cosmological solutions at intermediate times, that is at low redshifts, which is the period during the transition from matter to dark-energy domination. Such a solution can be very useful in dark energy observations, since probes based on supernovae Ia, WMAP and SDSS, are related to this cosmological epoch [1]. The comparison with the exact evolution shows that these first-order solutions are accurate within 2% up to $z \approx 0.5$.

In order to use the aforementioned results in a relative simple way, we have further approximated them extracting two new w-parametrizations, one as a function of the redshift z and one as a function of Ω_{DE}. Although the fitting parameters can be expressed as a function of the potential ones, one can equivalently consider them as free parameters suitably chosen to describe an arbitrary evolution. Due to the relative large number of free parameters, such parametrizations are very accurate up to present epoch. Thus one can use them in order to describe various quintom evolution sub-classes, including quintessence-like or phantom-like cases, realization of the -1-crossing, non-monotonic $w(z)$ evolution etc.

The above analysis shows that a two-field (one canonical and one phantom) quintom scenario with power-law potentials can offer a good description of dark energy evolution, in agreement with observations [1]. Moreover, the fact that power-law potentials can be justified through supersymmetric considerations [10] is an additional advantage. However, as it is usual in models where phantom fields are present, the quantum behavior of the examined scenario needs some caution, since the discussion about the construction of quantum field theory of phantoms is still open in the literature. For instance in [15] the authors reveal the causality and stability problems and the possible spontaneous breakdown of the vacuum into phantoms and conventional particles, but on the other hand in [16] the phantom fields arise as an effective description, consistently with the basic requirements of quantum field theory. The comparison with observations and a robust theoretical justification should be the decisive tests for the present model.

[1] A.G. Riess *et al.* [Supernova Search Team Collaboration], Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Astrophys. J. **517**, 565 (1999); A. Melchiorri *et al.*, Astrophys. J. Lett. **536**, L63 (2000); D. N. Spergel *et al.*, Astrophys. J. Suppl. **148**, 175 (2003); Bridle, O. Lahab, J. P. Ostriker and P. J. Steinhardt, Science **299**, 1532 (2003); J. L. Tonry *et al.*, Astrophys. J. **594**, 1 (2003); S. W. Allen, *et al.*, Mon. Not. Roy. Astron. Soc. **353**, 457 (2004).

[2] P. Binétruy, C. Deffayet, D. Langlois, Nucl. Phys. B **565**, 269 (2000); G.R. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B **485**, 208 (2000); S. Capozziello, Int. J. Mod. Phys. D **11**, 483 (2002); S.Nojiri and S. D. Odintsov, Phys. Rev. D **68**, 123512 (2003); A. Lue and G. Starkman, Phys. Rev. D **67**, 064002 (2003); P. S. Apostolopoulos, N. Brouzakis, E. N. Saridakis and N. Tetradis, Phys. Rev. D **72**, 044013 (2005); G. Calcagni, S. Tsujikawa and M. Sami, Class. Quant. Grav. **22**, 3977 (2005); S.Nojiri and S. D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007); R. P. Woodard, Lect. Notes Phys. **720**, 403 (2007); F. K. Diakonos and E. N. Saridakis, JCAP **0902**, 030 (2009).

[3] P. J. E. Peebles and B. Ratra, Astrophys. J. **325**, L17 (1988); B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988).

[4] C. Wetterich, Nucl. Phys. B **302**, 668 (1988); M. S. Turner and M. White, Phys. Rev. D **56**, 4439 (1997); R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998); Z. K. Guo, N. Ohta and Y. Z. Zhang, Mod. Phys. Lett. A **22**, 883 (2007); O. Hrycyna and M. Szydlowski, Phys. Rev. D **76**, 123510 (2007); S. Dutta, E. N. Saridakis, R. J. Scherrer, arXiv:0903.3412 [astro-ph.CO].

[5] I. Zlatev, L. Wang, and P. J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999); A. R. Liddle and R. J. Scherrer, Phys. Rev. D **59**, 023509 (1999); P. J. Steinhardt, L. Wang, and I. Zlatev, Phys. Rev. D **59**, 123504 (1999).

[6] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002); R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003); V. K. Onemli and R. P. Woodard, Phys. Rev. D **70**, 107301 (2004); P. F. Gonzalez-Diaz and C. L. Siguenza, Nucl. Phys. B **697**, 363 (2004); M. Sami, A. Toporensky, P. V. Tretjakov and S. Tsujikawa, Phys. Lett. B **619**, 193 (2005); A. Vikman, Phys. Rev. D **71**, 023515 (2005); Z. K. Guo, R. G. Cai and Y. Z. Zhang, JCAP **0505**, 002 (2005); S. Nojiri and S. D. Odintsov, Phys. Rev. D **72**, 023003 (2005); H. Garcia-Compean, G. Garcia-Jimenez, O. Obregon, and C. Ramirez, JCAP **0807**, 016 (2008); E. N. Saridakis, arXiv:0811.1333 [hep-th]; M. Szydlowski and O. Hrycyna, JCAP **0901**, 039 (2009); X. m. Chen, Y. Gong and E. N. Saridakis, arXiv:0812.1117 [gr-qc].

[7] B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B **607**, 35 (2005); Z. K. Guo, *et al.*, Phys. Lett. B **608**, 177 (2005); H. Wei, R. G. Cai and D. F. Zeng, Class. Quant. Grav. **22**, 3189 (2005); M.-Z Li, B. Feng, X.-M Zhang, JCAP, **0512**, 002 (2005); P. x. Wu and H. w. Yu, Int. J. Mod. Phys. D **14**, 1873 (2005); R. Lazkoz and G. Leon, Phys. Lett. B **638**, 303 (2006); Y. f. Cai, H. Li, Y. S. Piao and X. m. Zhang, Phys. Lett. B **646**, 141 (2007); W. Zhao and Y. Zhang, Phys. Rev. D **73**, 123509 (2006); B. Feng, M. Li, Y.-S. Piao and X. Zhang, Phys. Lett. B **634**, 101 (2006); M. R. Setare and E. N. Saridakis, Phys. Lett. B **668**, 177 (2008); M. R. Setare and E. N. Saridakis, JCAP **0809**, 026 (2008); M. R. Setare and E. N. Saridakis, Phys. Lett. B **671**, 331 (2009).

[8] P. S. Apostolopoulos, and N. Tetradis, Phys. Rev. D **74**, 064021 (2006); H.-S. Zhang, and Z.-H. Zhu, Phys. Rev. D **75**, 023510 (2007); Y. Gong and A. Wang, Phys. Lett. B **652**, 63 (2007); E. N. Saridakis, P. F. Gonzalez-Diaz and C. L. Siguenza, arXiv:0901.1213 [astro-ph].

[9] R. J. Scherrer and A. A. Sen, Phys. Rev. D **77**, 083515 (2008); R. J. Scherrer and A. A. Sen, Phys. Rev. D **78**, 067303 (2008); M. R. Setare and E. N. Saridakis, Phys. Rev. D **79**, 043005 (2009).

[10] P. Binetruy, Phys. Rev. D **60**, 063502 (1999); A. Masiero, M. Pietroni and F. Rosati, Phys. Rev. D **61**, 023504 (2000).

[11] J. P. Kneller and L. E. Strigari, Phys. Rev. D **68**, 083517 (2003); L. R. W. Abramo and F. Finelli, Phys. Lett. B **575**, 165 (2003); X. Zhang, Mod. Phys. Lett. A **20**, 2575 (2005); M. Yashar, B. Bozek, A. Abrahamse, A. J. Albrecht and M. Barnard, arXiv:0811.2253 [astro-ph].

[12] C. R. Watson and R. J. Scherrer, Phys. Rev. D **68**, 123524 (2003).

[13] E. N. Saridakis, arXiv:0902.3978 [gr-qc].

[14] G. Efstathiou, Mon. Not. R. Astron. Soc. **310**, 842 (1999); A. R. Cooray and D. Huterer, Astrophys. J. **513**, L95 (1999); M. Chevallier and D. Polarski, Int. J. Mod. Phys. D **10**, 213 (2001); E. V. Linder, Phys. Rev. Lett. **90**, 091301 (2003). E. Di Pietro and J. F. Claeskens, Mon. Not. Roy. Astron. Soc. **341**, 1299 (2003); M. Szydlowski, A. Kurek and A. Krawiec, Phys. Lett. B **642**, 171 (2006); E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006); H. Wei and S. N. Zhang, Phys. Lett. B **644**, 7 (2007).

[15] J. M. Cline, S. Jeon and G. D. Moore, Phys. Rev. D **70**, 043543 (2004).

[16] S. Nojiri and S. D. Odintsov, Phys. Lett. B **562**, 147 (2003); S. Nojiri and S. D. Odintsov, Phys. Lett. B **571**, 1 (2003).